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Modeling of supersonic plasma flow in the scrape-off layer

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Abstract

The substantial drop of the plasma temperature along magnetic field lines with increasing plasma density is one of the main features in tokamak divertors. As a result the temperature gradient at the divertor plates becomes very steep and the boundary condition normally applied for the parallel Mach number M at the target, $M_t = 1$, cannot be satisfied. In this case the value of M_t based on the general form of the Bohm criterion, $M_t \geqslant 1$, has to be determined from the continuity of plasma parameters.

In the present paper a new approach to resolve the Mach number at the target for such a situation is proposed. This method avoids the singularity problem that arises by treating the particle balance and parallel motion equations in a differential form. Instead, the integral representation of the equations is formulated for an arbitrary form of particle and momentum sources. The approach can also take into account transport perpendicular to the magnetic field lines.

The proposed method is demonstrated on the example of a one-dimensional stationary model for the scrape-off layer (SOL) plasma and includes the continuity-, parallel momentum- and heat transfer equations. The recycled neutrals are described in the diffusion approximation. In the case of low density the normal condition $M_t = 1$ is satisfied and the results are in agreement with the two-point model. At high enough plasma density solutions with the supersonic flow at the divertor plates, $M_t > 1$, are found. These states correspond to a partially detached plasma with a temperature of a few eV.

Keywords: Bohm criterion; Mach number; Supersonic flow; Scrape-off layer

1. Introduction

The operational scenario of fusion devices close to the density limit [1] is one of the most challenging tasks for theoretical modeling. Under these conditions the physical processes inside the scrape-off layer (SOL) and the divertor volume significantly affect the heat and particle transport in the core and the overall discharge performance. The favorable detached operation regime exists only in the presence of strong pressure gradients along the magnetic field in the SOL and leads to increased emission of neutral particles from the divertor volume [2]. Therefore, the predictive modeling of the edge plasma, which has experienced very significant progress

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during past years from two-point models [3,4] and the onion-skin method [5] to two- and three-dimensional simulations [6–9], remains an extremely important issue in fusion research.

One of the most critical points is the choice of boundary conditions for the transport equations, in particular, for the Mach number of the parallel plasma flow at the target plates, M_t. The Bohm criterion [10], derived from the stability of the sheath region near the surface, specifies only the lowest value: $M_t \geqslant 1$. Because of this uncertainty, the minimum level, $M_t = 1$, is often assumed by solving the momentum balance equation for plasma particles. However, as it has been shown in numerical calculations [11] and analytically for some simplified cases [3,12], the requirement $M_t = 1$ can be in contradiction with the transport equations themselves, in particular, if the plasma temperature gradient is strong enough near the target plate. Such a situation arises with increasing plasma density in the tokamak and the plasma temperature near the plate drops to a critical value. Indeed, the pressure gradient accelerates the plasma flow towards the target plates. On the other hand, plasma looses its momentum due to the friction force caused by the charge-exchange recombination with the recycled neutrals. For a hydrogen plasma these forces balance each other at a critical temperature of 3–4 eV.

In some transport codes for the plasma edge a fixed value of M_t is imposed as one of the boundary conditions and the plasma parameters are computed by integrating the transport equations from the plate towards the SOL main part. In the case of a parallel motion equation of the first order, i.e., without parallel viscosity, one may try to find the M_t value using an iterative procedure. In a situation requiring $M_t > 1$ this approach is, however, numerically unstable [13]. Physically this is due to the fact that the information on the boundary condition, spreading from the plate with the sound velocity, cannot propagate upstream of a supersonic flow [14]. Therefore in Ref. [11] another approach has been applied where the position x_s inside the plasma with $M = 1$ is determined from the requirement of the continuity of parallel velocity. The transport equations were integrated from x_s in both directions, upstream the subsonic flow toward the SOL and downstream the supersonic flow toward the plate. This method required a tricky expansion of the plasma parameters in the vicinity of the sonic transition point.

The numerical problems arising from a boundary condition taken in the form $M_t \geqslant 1$ can be avoided by introducing the parallel ion viscosity and increasing the order of the motion equation [15]. This approach has, however, its own pitfalls. First, the viscosity should be reduced dramatically near the plate where the parallel plasma velocity v varies very fast in space and exceeds the thermal one [16]. Normally this is not taken into account in edge plasma modeling. Second, the boundary condition assumed in the form $\mathrm{d}v/\mathrm{d}x = 0$ [3] is questionable. It is, in particular, in contradiction to the condition of the sheath stability requiring strong electric field and thus ion acceleration. The arbitrariness of this boundary condition results in an uncertainty of the solution of all transport equations.

A new approach to treat the situation with a supersonic flow near divertor plates is proposed in this paper. The particle- and momentum balance equations are analyzed in their integral form. With such a representation the Mach number at the target M_t, satisfying the Bohm criterion, is obtained from the requirement that a certain function achieves its maximum at the sonic transition point with $M = 1$. Moreover, the flow in the sub- and supersonic region corresponds to different branches of the solution of the algebraic equation.

Thus, the singularity problem appearing in the differential form of the transport equations is avoided. Up to now only the stationary one-dimensional solutions are analyzed and the parallel plasma viscosity is completely ignored. However, the generalization on non-stationary conditions with flows perpendicular to the magnetic field can be done straightforwardly.

A brief description of the model and basic equations are presented in the next section. The condition for supersonic flow in the SOL is formulated in Section 3. The approach to the solution of the motion equation and an algorithm for the determination of the parallel Mach number at the target are described in Section 4. Section 5 contains an example of the transition to detachment in the divertor. The comparison between our and previous calculations is summarized in Section 6. Concluding remarks are formulated in the last section.

2. Basic equations

The SOL is assumed in a slab geometry as schematically shown in Fig. 1. In the main part, from $x = 0$ to $x = x_d$, the SOL plasma is in contact with the confined plasma volume. Here, the particle and the heat fluxes

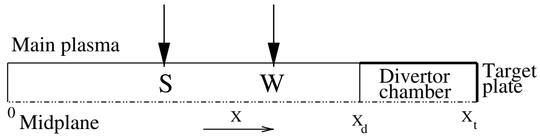

Fig. 1. The plasma edge geometry.

from the plasma core penetrate into the SOL, perpendicular to the magnetic field lines. The sources S and W corresponding to these fluxes are demonstrated in Fig. 1. In the SOL the plasma flow accelerates toward the divertor target. After recombination at the divertor plate, charged particles recycle as neutral atoms back into the SOL. Thus, equilibrium of the system is sustained by the escape of neutral particles from the divertor region, $x_d < x \leqslant x_t$, into the confined plasma volume. The launched power is lost by the plasma particles on the target plate and by ionization of recycling neutrals. A one-dimensional model of the SOL includes particle-, momentum balance- and heat transfer equations along the magnetic field lines. They describe the variation of the plasma density n, parallel velocity v and temperature T assumed to be the same for electrons and ions in the direction along field lines:

$$\frac{d}{dx}(nv) = S_p, \tag{1}$$

$$\frac{d}{dx}(mnv^2 + 2nT) = mvS_m, \tag{2}$$

$$\frac{d}{dx}\left(5nvT + \frac{mnv^3}{2} - \kappa_{\parallel}\frac{dT}{dx}\right) = S_E, \tag{3}$$

where m is the ion mass and the Spitzer formula is taken for the plasma heat conduction, $\kappa_{\parallel} = k_0 T^{5/2}$, with $k_0 \approx 2000$ W/m/eV$^{7/2}$. The particles, momentum and energy sources are given by the relations:

$$S_p = S\theta(x_d - x) + k_i n_n n - k_r n^2, \tag{4}$$

$$S_m = -n_n n k_{cx} - n^2 k_r, \tag{5}$$

$$S_E = W\theta(x_d - x) - k_i n_n n E_i - n^2 k_r\left(3T + \frac{mv^2}{2}\right) - n n_n k_{cx}\left(\frac{3}{2}T + \frac{mv^2}{2}\right). \tag{6}$$

Here, k_i, k_r and k_{cx} are the ionization, recombination (radiative and three-body) and charge-exchange rate coefficients, n_n is the density of recycling atoms and $E_i \simeq 30$ is the effective energy loss of electrons due to excitation and ionization of neutral atoms [17]. Ionization, radiative and three-body recombination rate coefficients are based on the Refs. [18–20] respectively. The charge-exchange rate coefficients were calculated by integrating cross-sections times relative velocity over a Maxwellian distribution [21]. The temperature of the neutral particles and ions was assumed to be equal. The function $\theta(x)$ is the Heaviside step function: $\theta(x \leqslant 0) = 0$, $\theta(x > 0) = 1$. The first term in (4) describes the source due to perpendicular diffusion of charged particles from the confined plasma, the second one – ionization of neutrals recycled from the target plate, and the last one – loss by recombination at low temperatures. The ion momentum losses in Eq. (5) include charge-exchange with neutrals and plasma recombination. We note that the neutral particles penetrate into the plasma perpendicular to the target plates. Since the pitch angle α between magnetic field lines and poloidal divertor is small the component of their velocity relative to the velocity of ions along the magnetic field lines is negligible. Eq. (6) includes the energy source W caused by the heat flux from the main plasma volume and plasma-neutral energy exchange by ionization, recombination and charge-exchange.

An adequate description of recycling neutral particles, especially at low plasma temperatures and high densities, requires kinetic models [22,23] or Monte-Carlo simulations [24]. Indeed, at such conditions the plasma profiles are very inhomogeneous near the target plate. As a consequence, the mean free path of neutral particles is longer than the characteristic length of the change of the plasma parameters. Nevertheless, for our qualitative analysis we use a fluid diffusion approximation, which has been often applied earlier [4,25] and which gives a relatively good agreement with the experimental measurements, see Ref. [26]:

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(-D_n\frac{\mathrm{d}n_n}{\mathrm{d}z}\right) = -k_\mathrm{i}n_nn + k_\mathrm{r}n^2 - \frac{n_n}{\tau_n}\theta(x_\mathrm{d} - x). \tag{7}$$

Here $z = (x_\mathrm{t} - x)\sin\alpha$ is the distance from the target plate and $D_n = T/m/n(k_\mathrm{i} + k_\mathrm{cx})$ is the diffusion coefficient of neutral particles. The last term in Eq. (7) describes the outflow of neutrals into the confined volume. It balances the particle flux from the main plasma and permits, therefore, the steady state solution of the equations. The characteristic escape time τ_n is equal to v_n/Δ, where v_n is the neutral velocity and Δ is the characteristic radial width of the SOL. It is assumed that neutrals are reflected back into the plasma at the outer boundary of the SOL.

In the present approximation we do not consider the parallel neutral momentum transfer, and the parallel velocity of neutrals is neglected compared to the ion speed v. However, neutrals acquire parallel momentum after charge-exchange recombination. As a result the relative velocity between ions and neutrals, and thus the friction force on ions, is smaller than assumed above. This should, in particular, lead to a higher temperature for the transition to supersonic flow than obtained in this paper.

The boundary conditions for Eqs. (1)–(3) and (7) are imposed by the symmetry of the system at the mid-plane $x = 0$:

$$M = \mathrm{d}T/\mathrm{d}x = \mathrm{d}n_n/\mathrm{d}z = 0. \tag{8}$$

At the target plate the Bohm criterion provides the boundary condition for the Mach number:

$$M \geqslant 1. \tag{9}$$

The continuity of the heat flux imposes the boundary condition for the plasma temperature and its gradient in the sheath:

$$-\kappa_\parallel \mathrm{d}T/\mathrm{d}x + 5nvT = \gamma nvT, \tag{10}$$

with $M \equiv v/c_\mathrm{s}$ being the Mach number, $c_\mathrm{s} = \sqrt{2T/m}$ is the ion sound velocity and $\gamma \approx 7.5$ is the heat transmission coefficient to the plate [3]. Finally, the flux of the neutral particles follows the condition of ideal recycling at the target plates:

$$-D_n\mathrm{d}n_n/\mathrm{d}z = nv\sin\alpha. \tag{11}$$

3. Condition for supersonic flow in SOL

Since the Bohm criterion (9) is an inequality, the lowest value $M = 1$ at the target is often assumed in SOL simulations. As one can easily demonstrate this is indeed the only possibility if the plasma temperature near the target plate is high. Thus, by combining Eqs. (1) and (2), the following differential equation for the Mach number can be obtained:

$$nc_\mathrm{s}(1 - M^2)\frac{\mathrm{d}M}{\mathrm{d}x} = R \equiv M^2(S_\mathrm{p} - S_\mathrm{m}) + S_\mathrm{p} + \frac{nc_\mathrm{s}M(1 + M^2)}{2T}\frac{\mathrm{d}T}{\mathrm{d}x}. \tag{12}$$

Here, the momentum source term S_m is always negative. If the plasma temperature is not significantly lower than the hydrogen ionization potential, the charged particle source is dominated by ionization of neutrals and particle source S_p is positive. Besides, the electron parallel heat conduction is strong and the temperature gradient and thus the last term in the right hand side (RHS) is small. As a result the RHS is positive in the whole plasma region, $0 \leqslant x \leqslant x_\mathrm{t}$. In such a case, in close vicinity of the point x_s with $M = 1$, Eq. (12) can be approximated by the following equation:

$$(1 - M)\frac{\mathrm{d}M}{\mathrm{d}x} \simeq A = \frac{R(x_\mathrm{s})}{2nc_\mathrm{s}} > 0. \tag{13}$$

By integrating this equation with the boundary condition $M(x_\mathrm{s}) = 1$, one obtains:

$$M - \frac{M^2}{2} \simeq A(x - x_\mathrm{s}) + \frac{1}{2} \tag{14}$$

and

$$M \simeq 1 \pm \sqrt{2A(x_s - x)}. \tag{15}$$

First, in the case in question for M increasing with x only the solution with sign $(-)$ is relevant. Second, there is no any real solution for $x > x_s$. Thus x_s should be the largest possible x, i.e., it can correspond to the target plate position only.

Now consider the case of temperatures near the plate being much lower than the ionization potential. First, the particle source term S_p in Eq. (4) becomes negative as the recombination is now the dominating process. Second, the plasma heat conductivity becomes low and a large negative temperature gradient develops near the plate. As a result the RHS in Eq. (12) becomes also negative at target plate $x = x_t$. Assuming that $M_t = 1$, no real solution for Mach number M is possible for $x < x_s = x_t$, since $A < 0$ in Eq. (15). Therefore, the Mach number at the target should be larger than one: $M_t > 1$. This implies that point x_s with $M = 1$ is located within the plasma. Moreover, the RHS of Eq. (12) is equal to zero at this point, as $M(x)$ is a continuous function. Indeed, since the plasma temperature increases by going from the plate towards the SOL main part, the source S_p increases and the RHS becomes positive. At the position $x = x_s$ the RHS changes sign, i.e., $R \approx \mathrm{d}R/\mathrm{d}x(x_s) \times (x - x_s)$. Since $M \approx 1$ Eq. (12) can be approximated as follows:

$$(1 - M)\frac{\mathrm{d}M}{\mathrm{d}x} \simeq -B(x - x_s) \tag{16}$$

with $B = -\frac{1}{2nc_s}\frac{\mathrm{d}R}{\mathrm{d}x}(x_s) > 0$. The solution for Mach number M increasing with x is given by:

$$M \simeq 1 + \sqrt{B}(x - x_s). \tag{17}$$

Physically, the temperature gradient accelerates the plasma flow from the midplane towards the target and if this gradient is too strong the value $M = 1$ can be achieved before the target. One can estimate roughly when this happens by assuming the RHS of Eq. (12) is equal to zero at the plate. The temperature gradient and neutral density can be evaluated from the boundary conditions (10) and (11). This results in the relation:

$$(2k_i + k_{cx})\sin\alpha = (\gamma - 5)\frac{c_s^2}{\kappa_\parallel}. \tag{18}$$

For $\alpha = 0.05$ one gets $T_t \equiv T(x_t) < 4$ eV as the condition necessary for the supersonic flow to the plate.

4. Approach for numerical treatment of supersonic flow in SOL

The procedure applied in Ref. [11] was based on direct integration of Eqs. (1), (3) and (12) in the following way. The RHS of Eq. (12) was computed with the assumed profiles for the density $n(x)$, Mach number $M(x)$ and temperature $T(x)$. Near the SOL symmetry plane, $x = 0$, $\mathrm{d}T/\mathrm{d}x$ is small and the RHS is positive. For standard situations with $M_t = 1$ it is also positive near the target. If, however, this is not the case a supersonic flow should be expected in this region otherwise the Bohm condition cannot be satisfied. In order to presume the continuity of Mach number one has to assume that $M = 1$ at the point x_s where the RHS changes its sign. From this point Eq. (12) can be integrated in both directions towards the main part of the SOL and towards the target plate, in order to find the new approximation for $M(x)$. However, as it was demonstrated in the previous section, in order to start this integration one has to find the derivative of the RHS at the point x_s. This procedure is not fully straightforward and can provide a large error.

In this paper we propose another approach by using an integral equivalent of Eq. (12). Moreover, this method allows to determine the Mach number at the target plate M_t and can be used in codes operating with parallel viscosity. If some approximation to the solution of the system of Eqs. (1)–(3) is assumed, the continuity equation (1) can be integrated providing the particle flux density:

$$nc_sM = n_tc_{st}M_t - \int_x^{x_t} S_p\,\mathrm{d}x', \tag{19}$$

where c_{st} is the ion sound velocity at the target plate. The boundary condition $M = 0$ at the symmetry plane with $x = 0$, requires:

$$n_tc_{st}M_t = \int_0^{x_t} S_p\,\mathrm{d}x'. \tag{20}$$

By integrating the momentum equation (2), we obtain

$$nc_s^2(1+M^2) = n_t c_{st}^2(1+M_t^2) - \int_x^{x_t} mvS_m\,dx'. \tag{21}$$

In the latter $n(x)$ and n_t can be expressed through other variables by using Eqs. (19) and (20), and one obtains:

$$\begin{aligned}
&\frac{M(x)}{1+M^2(x)} = F(x,M_t) \equiv \frac{M_t G_p(x)}{1+M_t^2 G_m(x)} \times \sqrt{\frac{T(x)}{T_t}},\\
&G_p(x) = 1 - \frac{1}{n_t c_{st} M_t}\int_x^{x_t} S_p\,dx',\\
&G_m(x) = 1 - \frac{1}{n_t c_{st} M_t^2}\int_x^{x_t} \sqrt{\frac{T}{T_t}} M S_m\,dx'.
\end{aligned} \tag{22}$$

Eq. (22) can be resolved as a quadratic one in order to obtain the Mach number profile:

$$M_\pm(x) = \frac{1}{2F(x)} \pm \sqrt{\frac{1}{4F^2(x)} - 1}. \tag{23}$$

These solutions include, however, M_t as unknown parameter. In order to find the Mach number at the target Eq. (22) has to be analyzed in details.

The left side (LHS) of Eq. (22) approaches its maximum value 1/2 at a point where the Mach number $M = 1$. Consider now the behavior of the RHS of the same equation. At high plasma temperature the recombination can be neglected in the particle source. At this condition the source term $S_p > 0$ and $G_p(x)$ decreases monotonically from 1 to 0 by moving away from the target. Since momentum source S_m is negative, $G_m(x)$ increases from its minimum value 1 at the target with decreasing x. Thus, the first factor in $F(x, M_t)$ decreases by moving from the target. If the temperature variation is weak, the decay of the first factor is dominating in the RHS of Eq. (22) and the maximum value of F is achieved already at the target and is equal to $F_{max} = \frac{M_t}{1+M_t^2}$. Since F_{max} has to be the same as the maximum of the LHS, 1/2, it follows that $M_t = 1$. One can see that only the solution $M_-(x)$ can satisfy simultaneously to this requirement and to the boundary condition at the symmetry plane, $M\,(x = 0) = 0$. If, however, the temperature increases fast enough with decreasing x, then function F approaches its maximum value F_{max} at some point $x_* < x_t$ and F_{max} depends on M_t. This is demonstrated in Fig. 2 by $F(x, M_t)$ computed for the conditions described in the next section. The requirements

$$F(x_*) = 1/2 \tag{24}$$

and

$$dF/dx(x_*) = 0 \tag{25}$$

defines both x_* and M_t. First, as it follows from Eq. (23), $M(x_*) = 1$, i.e., x_* is the position of the sonic transition. Second, one can find that:

$$\frac{dF}{dx} = F \times \left(\frac{1}{G_p}\frac{dG_p}{dx} - \frac{M_t^2}{1+M_t^2 G_m}\frac{dG_m}{dx} - \frac{1}{2T}\frac{dT}{dx}\right). \tag{26}$$

From Eq. (24) we obtain

$$\frac{M_t^2}{1+M_t^2 G_m} = \frac{M_t}{2G_p}\sqrt{\frac{T_t}{T}} \tag{27}$$

and, by using the definitions of S_p, S_m and Eqs. (19) and (20), Eq. (25) is reduced to the following equation for x_*:

$$2S_p(x_*) - S_m(x_*) = \frac{nc_s}{T}\frac{dT}{dx}(x_*). \tag{28}$$

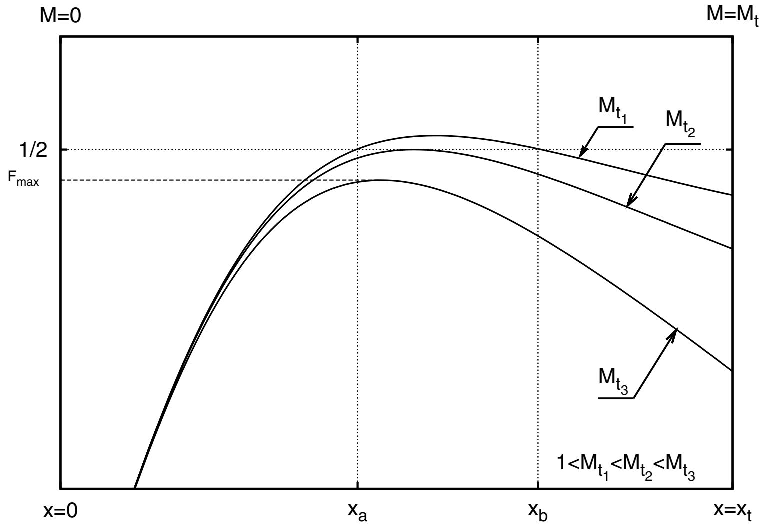

Fig. 2. The RHS of Eq. (22), $F(x, M_t)$, as a function of x for different M_t values.

One can see that this equation coincides with the requirement of zero RHS of Eq. (12) in the sonic point. If x_* is found, M_t can be determined by solving Eq. (24) for $M_t > 1$:

$$M_t = \frac{G_P(x_*)}{G_m(x_*)}\sqrt{\frac{T(x_*)}{T_t}} + \sqrt{\left[\frac{G_P(x_*)}{G_m(x_*)}\right]^2 \frac{T(x_*)}{T_t} - 1}. \tag{29}$$

The total profile of the Mach number is given by M_- at $0 \leqslant x \leqslant x_*$ and by M_+ at $x_* \leqslant x \leqslant x_t$. Only this combination satisfies the boundary conditions $M\ (x = 0) = 0$ and $M\ (x = x_t) = M_t > 1$. If M_t is not equal to the value given by Eq. (29) and $F_{max} \neq 1/2$, it is easy to demonstrate that $M(x)$ cannot be real and continuous. Assume that M_t is too small and $F_{max} > 1/2$. Then there is no any real solution in the range $x_a < x < x_b$, where $x_{a,b}$ are defined by the conditions $F(x_{a,b}) = 1/2$. If however M_t is so large that $F_{max} < 1/2$, one has to choose again M_- at $0 \leqslant x \leqslant x_*$ and M_+ at $x_* \leqslant x \leqslant x_t$. Thus there is a jump in the Mach number at x_* equal to:

$$\delta M = M_+(x_*) - M_-(x_*) = \sqrt{1/F_{max}^2 - 4}. \tag{30}$$

Finally, the particle and momentum sources, S_p and S_m, have been selected without any restrictions and can include the divergence of the particle and momentum fluxes perpendicular to the magnetic field. Therefore the method proposed can be applied to two- or three-dimensional models.

5. Results of calculations

The approach proposed has been applied to investigate the transition to the detachment mode in the divertor by increasing the plasma density at the SOL symmetry plane $x = 0$. For this purpose the input power to the SOL W and the density at the midplane n_m or at the target n_t are considered as the input parameters to the model. In this case the source term S results from the calculations. The set of Eqs. (3), (19) and (23) for the determination of $T\ (x = x_t)$, $n\ (x = x_t)$ and $M\ (x = x_t)$, respectively, has been solved by iterations. The geometrical parameters correspond to the tokamak JET: the total connection length $x_t = 50$ m, the distance from the midplane up to the X-point $x_d = 45$ m, the pitch angle of the magnetic field $\alpha = 5°$, the radial width of the SOL $\Delta = 2$ cm. The heating power density due to heat conduction into the SOL from the confined plasma volume $W = 1$ MW/m^3. Fig. 3 shows the calculated n_m-dependence of the ion saturation current $n_t c_{st} M_t$, ion momentum $n_t T_t(1 + M_t^2)$ and Mach number at the target plate M_t.

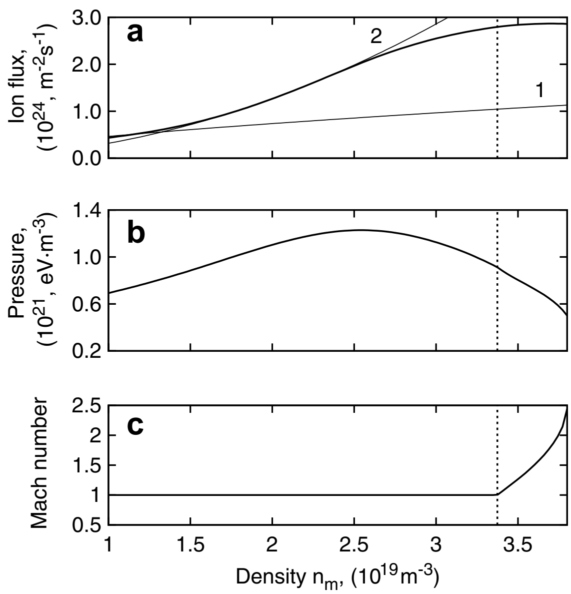

Fig. 3. Dependences of the ion saturation current (a), ion momentum (b) and Mach number (c) at the target on the midstream density n_m. The ion saturation current variation predicted by the two-point model for the sheath-limited and high-recycling regime is given by curves 1 and 2, respectively.

The SOL heat transport is dominated by the convection for midplane densities below 1.2×10^{19} m^{-3} and the ion saturation current scales as $n_m^{2/3}$ (curve 1). This regime is referred to as the sheath-limited one. At higher densities, up to about 2.5×10^{19} m^{-3}, the SOL plasma is in the high recycling regime, where the parallel heat conduction dominates, and the ion saturation current scales as n_m^2 (curve 2). For these regimes our results are in agreement with the two-point model [3].

The transition from the high-recycling regime to partial detachment from the target is characterized by the deviation of the latter dependence and by the drop of the total momentum. This happens at a critical n_m being higher than 2.5×10^{19} m^{-3}. At this transition, however, the plasma is still in the subsonic region. The transition to the supersonic flow occurs at the density of 3.4×10^{19} m^{-3} and the temperature of 3 eV at the target. It is interesting to note, that at this transition the Mach number at the target increases almost linearly with n_m, as it was also obtained in Ref. [11]. When T_t drops below 1 eV the density dependence of M_t becomes significantly non-linear. The profiles of the plasma parameters computed for $n_m = 3.5 \times 10^{19}$ m^{-3} are demonstrated in Fig. 4. The plasma pressure starts to drop at the position of the ionization front where the particle source and momentum sink approach their maximum levels. After this position the plasma density decreases and the Mach number exceeds one. The plasma behavior for higher n_m has not been successfully modeled up to now because of numerical problems occurring when the plasma temperature near the plate drops significantly below 1 eV. At these temperatures the diffusion approximation does not reproduce the neutral particle losses into the confined volume. The corresponding development of the model will be done in future.

6. Method validation

In order to validate the method proposed and to illuminate its important features, we compare our results with those of Ref. [25] where the minimum Mach number at the target plate, $M_t = 1$, has been adopted. For this comparison impurity radiation has been included into the present model by adding the term $S_I = -\xi n^{2}$-$L_I(T)$ to the energy loss S_E, Eq. (6). Here $\xi = 0.05$ is the relative concentration of carbon impurity and L_I is the impurity rate computed according to the corona model [27]. In addition as in Ref. [25] the SOL has been assumed fully opaque for neutrals by taken $\tau_n = \infty$ in Eq. (7).

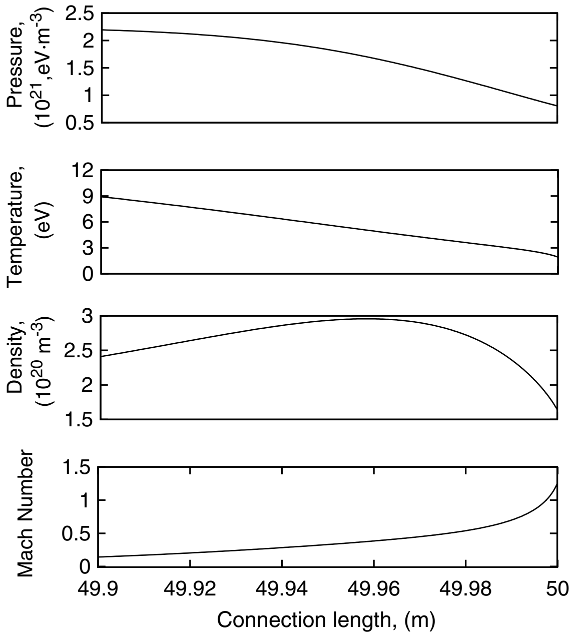

Fig. 4. Plasma parameter profiles computed for $n_m = 3.5 \times 10^{19}\ m^{-3}$.

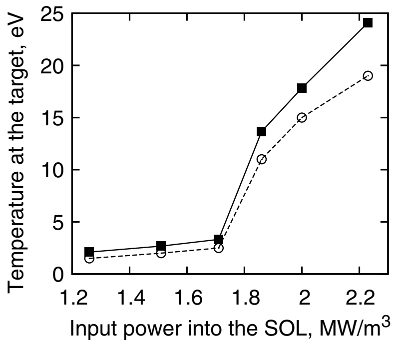

Fig. 5. The power density dependence of the plasma temperature near the target computed with the present model (solid line with squares) and taken from Ref. [25] (dashed line with circles).

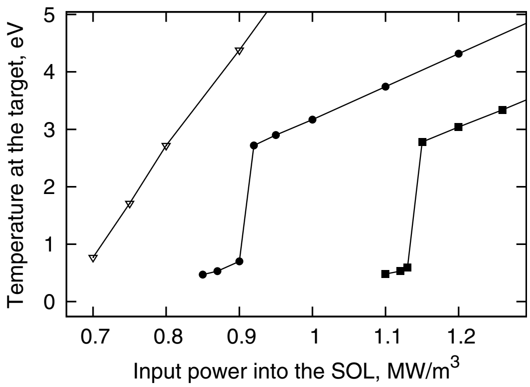

Fig. 6. The power density dependence of the plasma temperature near the target computed with the present model for different level of impurity radiation: $\xi = 0$ (solid line with triangles), $\xi = 0.01$ (solid line with circles), $\xi = 0.05$ (solid line with squares).

Fig. 5 shows the temperature at the target plate versus the input power density W. In our computation demonstrated by the solid curve the density at the target n_t was chosen the same as in Ref. [25]. One can see that for the power density range in question both models provide close results; the relatively small deviation is most probably due to differences in the data of atomic processes. No stationary solutions were found in Ref. [25] for an input power lower than the range in Fig. 5 and temperatures at the target below 2 eV. The model applied there allowed description of non-stationary states and under such conditions a front of cold plasma moving to the main part of the SOL was observed. The present approach, which takes into account the development of a supersonic flow with $M_t > 1$ provides stationary states even in this case with T_t down to 0.5 eV, see Fig. 6. The dependences here have been found by keeping the density at the midplane constant and equal to $2.24 \times 10^{13}\ \mathrm{cm}^{-3}$.

7. Conclusions

A new approach for the solution of the plasma equations in the SOL with a supersonic flow is proposed. This method avoids the consideration of the motion equation in the differential form and therefore the difficulties connected with the singularity at the sonic transition. For the given profiles of the particle source and momentum sink this position can be determined and the Mach number at the target plate can be calculated. The latter can be used as a boundary condition for the Mach number in the edge transport codes operating with ion parallel viscosity. The approach can be used with sources including the divergence of flux components perpendicular to the magnetic field, i.e., to be expanded to two- and three-dimensional transport models.

This method has been implemented into a stationary one-dimensional model for the SOL. The results of calculations are in agreement with a two-point model in the sheath-limited regime at low plasma density and in the high-recycling regime at the intermediate density. By transition to the partial detachment mode the temperature gradient accelerates the plasma to the sonic velocity inside the SOL and a region with a supersonic flow appears close to the target plate.

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